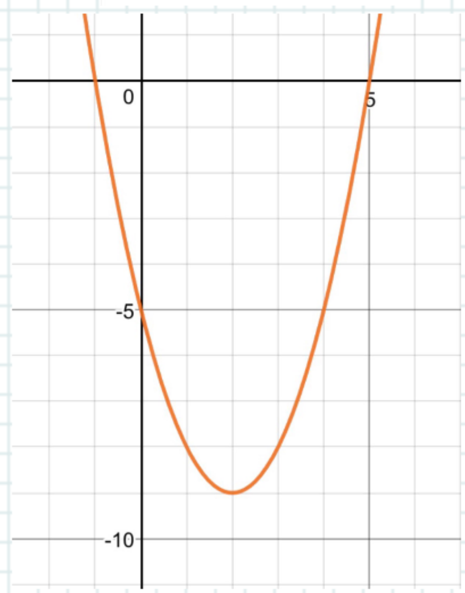
The discriminant

The quadratic equation

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

gives the two roots of a quadratic equation. The discriminant is the part under the radical, b^2-4ac. Even before we do the rest of the math, the discriminant offers details regarding the roots.

$y = x^2 - 4x - 5$



Here, $a=1$, $b=-4$, $c=-5$

$b^2-4ac = (-4)^2-(4)(1)(-5) = 36$

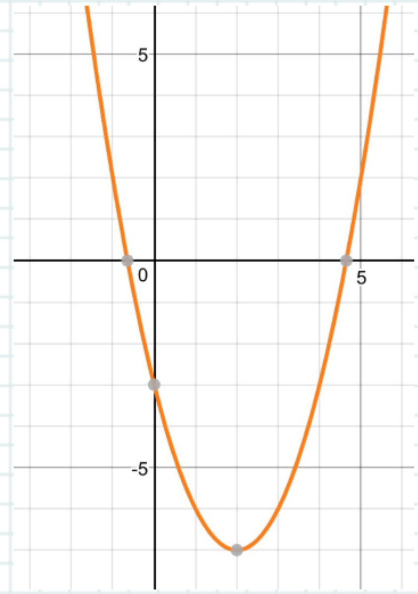
We see 36 is a perfect square, which tells us the roots are rational.

Now, let's see what happens when we change 'c' a bit.

$a = 1 \quad b = -4 \quad c = -3$

$b^2 - 4ac = (-4)^2 - (4)(1)(-3) = 28$

$y = x^2 - 4x - 3$



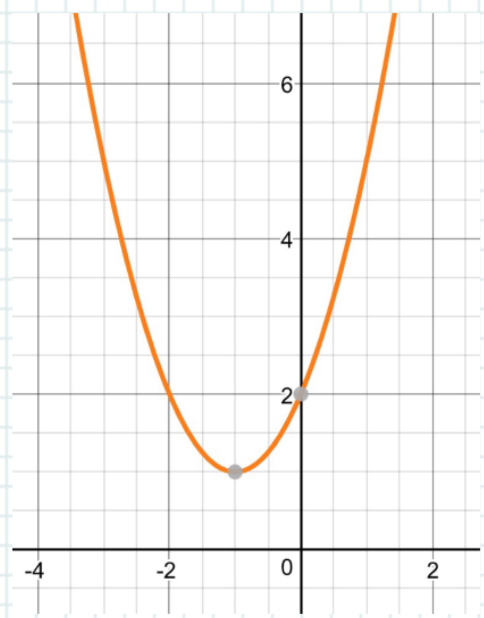
We still have 2 roots. They are real, but not rational.

$$x = \frac{4 \pm \sqrt{28}}{2} = 2 \pm \sqrt{7}$$

Next, $a = 1 \quad b = 2 \quad c = 2$

$b^2 - 4ac = 4 - 8 = -4$

$y = x^2 + 2x + 2$



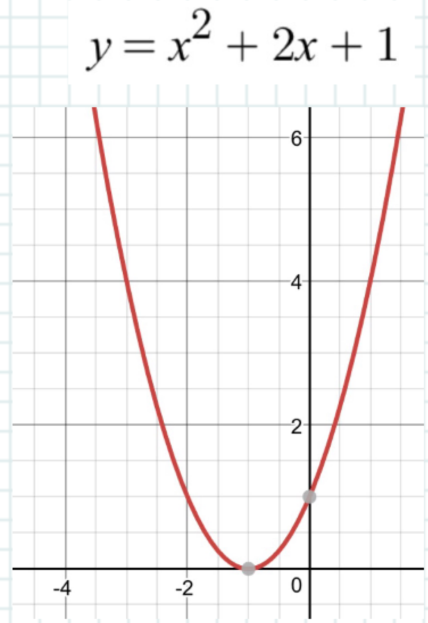
A negative discriminant tells us there's no real root. The graph doesn't intersect the x-axis. If you finish with the quadratic equation you find roots $-1 \pm i$, the solution is imaginary.

One last example.

$b^2 - 4ac = 2^2 - (4)(1)(1) = 0$

When the discriminant is zero, the vertex, $x = \frac{-b}{2a}$ results in $y = 0$, and one double root.

Here $x = -1$



The discriminant can have 4 results that describe the nature of the roots:

	Discriminant	Roots
①	negative	imaginary
②	zero	one double root
③	positive	2 real roots
④	positive, perfect square	2 rational roots