

The quadratic equation

$$ax^2 + bx + c = 0 \quad \text{divide both sides by 'a'}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad \text{set up to complete square}$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \quad \text{complete by squaring } \frac{1}{2} \text{ of second coefficient}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a} \quad \leftarrow \text{mult by } \frac{4a}{4a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \frac{\pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$