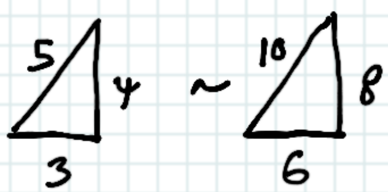
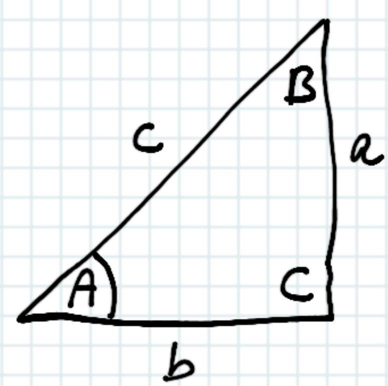
Intro to trig

We've learned about similar triangles, which have congruent angles and whose sides are in identical ratios.



Second triangle has sides that are twice the size of the first. Also, each triangle's sides are in the same ratio, eg the base is $\frac{3}{4}$ the length of the height. Trigonometry is based on this, the side ratios. We start with the basic 3 functions, Sine (sin), Cosine (cos), & Tangent (tan), these are 3 ratios comparing the lengths of 2 sides of a triangle.



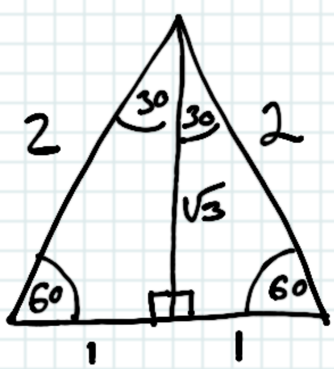
$$\sin(A) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos(A) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan(A) = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

Let's review the ratio for a 30°-60°-90° triangle and apply these ratios to the trig functions —

First, draw an equilateral triangle, and note the 2 base angles are 60°. Let's make the 2 sides 2 units long. Now, drop an altitude. We just bisected the top 60° angle to 30° each. And bisected the base to 2 1unit segments. What is the height? Pythag? $x^2+1^2=2^2$ $\quad x=\sqrt{3}$



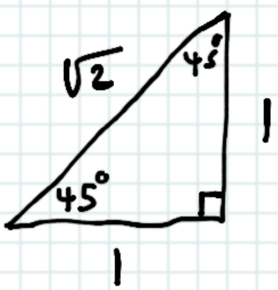
This is how to get the side ratios with no memorizing. All thats left is to get adjacent (it means 'touching') opposite (it's the side between the other two angles) and hypotenuse (the biggest side) all identifed correctly. So...

$\sin(30)=\frac{1}{2}$ $\quad \sin(60)=\frac{\sqrt{3}}{2}$

$\cos(30)=\frac{\sqrt{3}}{2}$ $\quad \cos(60)=\frac{1}{2}$

$\tan(30)=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$ $\quad \tan(60)=\sqrt{3}$

Next, the 45° right triangle



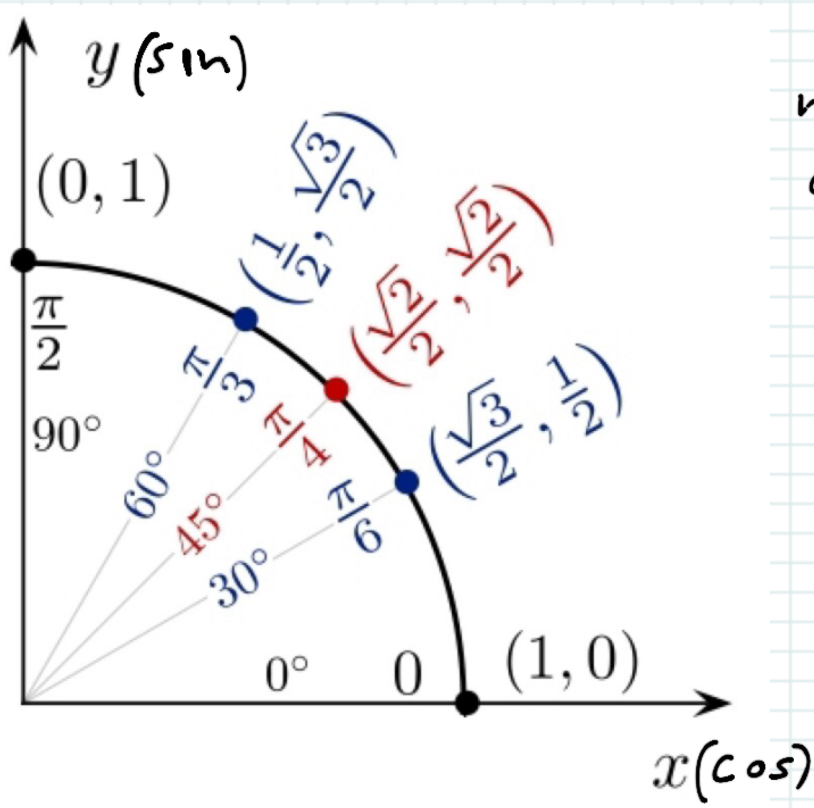
If the sides are 1 unit long, simple Pythag gives $x^2 = 1^2 + 1^2$ or $x = \sqrt{2}$

$\sin(45) = 1/\sqrt{2} = \frac{\sqrt{2}}{2}$ [You remember how to rationalize the denominator?]

$\cos(45) = 1/\sqrt{2} = \frac{\sqrt{2}}{2}$

$\tan(45) = 1/1 = 1$

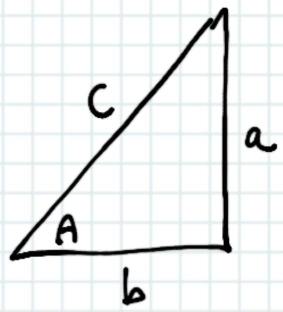
Now, you can get all 3 trig functions for 30°, 45°, & 60°. The easy way to remember the ratios is SOHCAHTOA



If we take the six numbers we calculated and graph, we see an arc of a circle.

See next page for proof it's a circle. This lets us fill in $\sin 0° = 0$ and $\sin 90° = 1$, which make less sense looking at triangles.

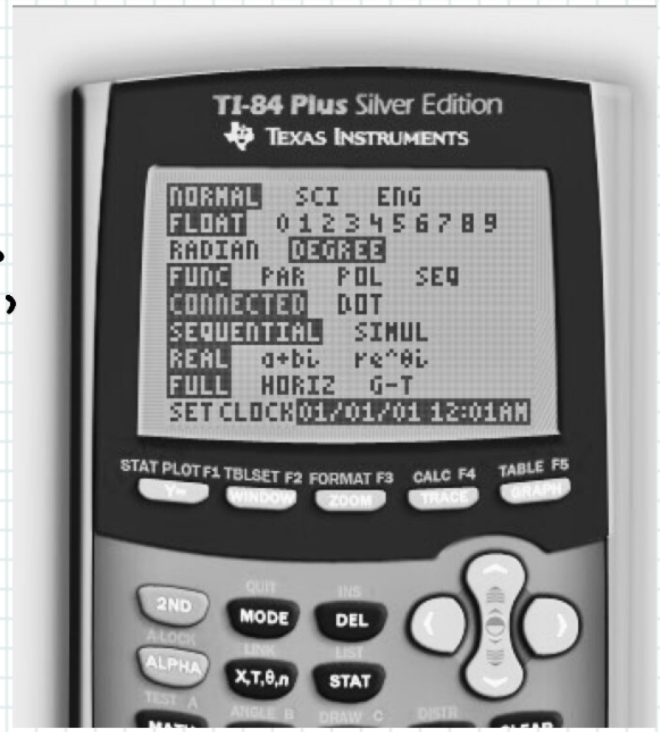
Proving the trig functions are circular



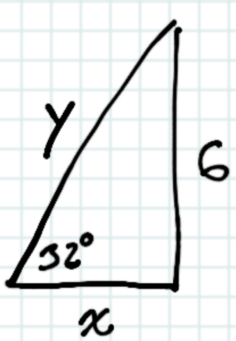
$$\sin A = \frac{a}{c} \quad \& \quad \cos A = \frac{b}{c}$$

so $$\sin^2 A + \cos^2 A = \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2+b^2}{c^2} = 1$$

Important note - make sure your calculator is set to degrees. Just → hit 'Mode', highlight 'Degree' on third row and 'enter.'



If we are given an angle, along with one side, we can calculate the missing sides. In this figure, we have O and want A and H. Since $\tan\theta = O/A$, $\tan 32° = \frac{6}{x}$ and

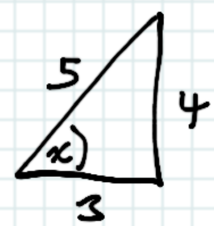


$$x = \frac{6}{\tan 32°} = \underline{9.6}$$

$$\sin\theta = \frac{O}{H} \qquad \sin 32° = \frac{6}{y}$$

$$y = \frac{6}{\sin 32°} = \underline{11.32}$$

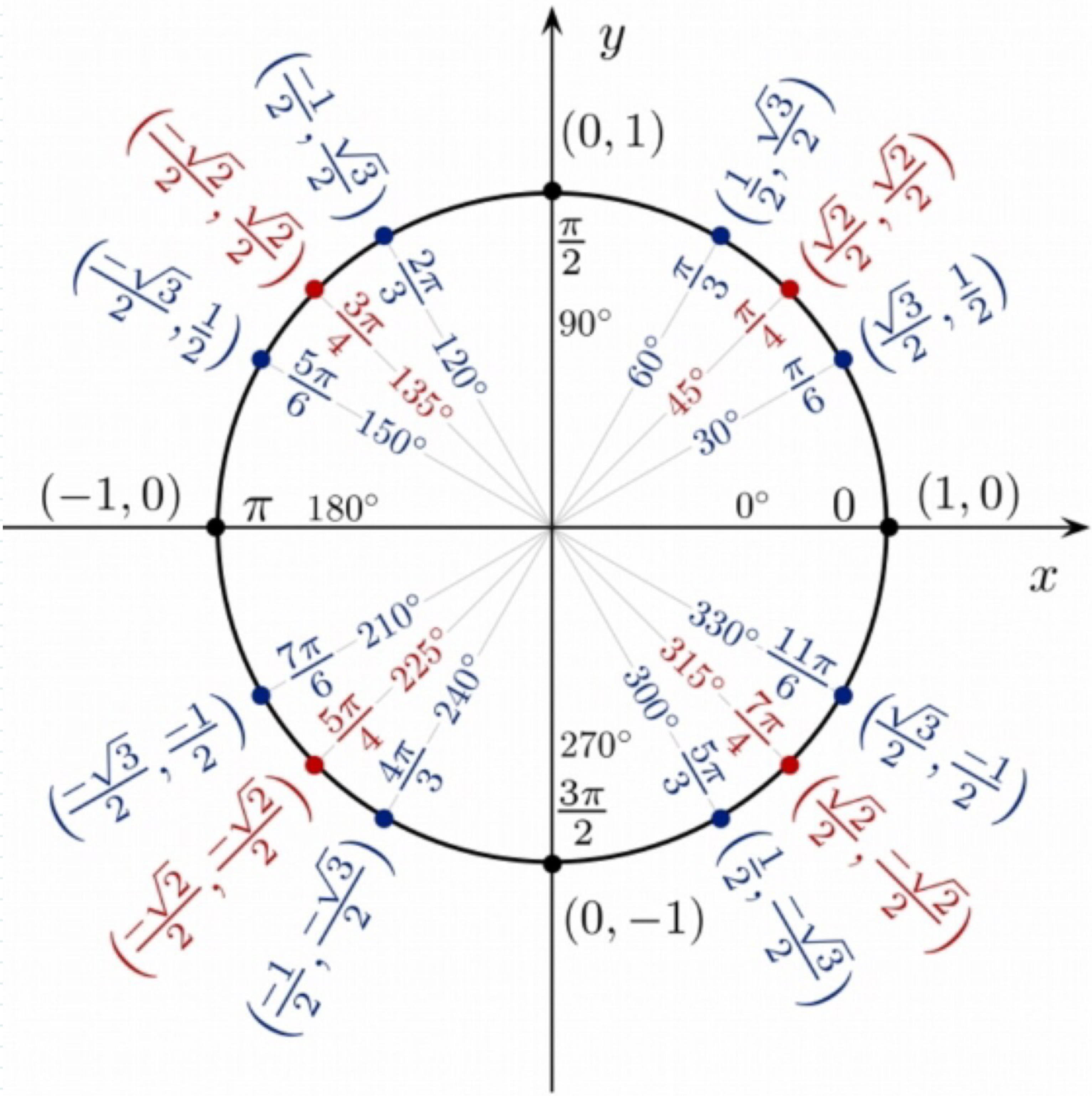
Keep in mind, if $\sin x = \frac{O}{H}$ we have 3 things, or rather, we have 2 things but want the third. Remember the 3-4-5 triangle?



$\sin x = 4/5 = .8$

here, we need $\sin^{-1}(.8) = 53.1^\circ$. This is an example of when you have the sides, but don't know the angle. The notation, $\sin^{-1}(.8)$ is the same as $\arcsin(.8)$ and it's the mathy way to say "tell me the angle whose sin is .8"

Last page is the full unit circle. Sine and Cosine go beyond the 90° right triangle and the unit circle will come in handy.



y
x
$(0,1)$
$(1,0)$
$(-1,0)$
$(0,-1)$
$\frac{\pi}{2}$
$90°$
π
$180°$
$0°$
0
$270°$
$\frac{3\pi}{2}$
$\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
$\frac{\pi}{3}$
$60°$
$\frac{\pi}{4}$
$45°$
$\frac{\pi}{6}$
$30°$
$\left(\frac{-1}{2},\frac{\sqrt{3}}{2}\right)$
$\left(\frac{-\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
$\left(\frac{-\sqrt{3}}{2},\frac{1}{2}\right)$
$\frac{2\pi}{3}$
$120°$
$\frac{3\pi}{4}$
$135°$
$\frac{5\pi}{6}$
$150°$
$\frac{7\pi}{6}$
$210°$
$\frac{5\pi}{4}$
$225°$
$\frac{4\pi}{3}$
$240°$
$\left(\frac{-\sqrt{3}}{2},\frac{-1}{2}\right)$
$\left(\frac{-\sqrt{2}}{2},\frac{-\sqrt{2}}{2}\right)$
$\left(\frac{-1}{2},\frac{-\sqrt{3}}{2}\right)$
$330°$
$\frac{11\pi}{6}$
$315°$
$\frac{7\pi}{4}$
$300°$
$\frac{5\pi}{3}$
$\left(\frac{\sqrt{3}}{2},\frac{-1}{2}\right)$
$\left(\frac{\sqrt{2}}{2},\frac{-\sqrt{2}}{2}\right)$
$\left(\frac{1}{2},\frac{-\sqrt{3}}{2}\right)$